Econ 802

Second Midterm Exam

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All questions have equal weight. If something is unclear, please ask.

1. Jack has preferences over two goods $(x_1, x_2) \geq 0$.

(a) Define local non-satiation. Then prove that if this assumption holds, an optimal bundle must be on the budget line, not below it. Explain briefly using a graph.

(b) Define strictly convex preferences. Then prove that if this assumption holds, the optimal bundle is unique. Explain briefly using a graph.

(c) Define the Marshallian demand vector x(p, m) where $(p_1, p_2, m) > 0$ and show that it is homogeneous of degree zero. You can use the assumptions in (a) and (b).

2. Jean has the utility function $u(x, y) = 1 - e^{-ax} + y$ where $x \geq 0$, $y \geq 0$, and $a > 0$.

(a) Putting x on the horizontal axis and y on the vertical axis, draw a few indifference curves. Is the utility function strictly quasi-concave? Why or why not?

(b) Let $p > 0$ be the price of x, let $q > 0$ be the price of y, and let $m > 0$ be income. Assuming the solution is interior, find some $(x^*, y^*) > 0$ that satisfies the first order conditions for utility maximization. Does the second order <u>necessary</u> condition hold? Does the second order <u>sufficient</u> condition hold? Explain.

(c) Taking account of the possibility that there may be boundary solutions for x or y, solve for the Marshallian demand functions for both goods for all $(p, q, m) > 0$. Carefully justify your reasoning.

3. Jim has the direct utility function $u(x_1, x_2) = \alpha \ln x_1 + (1-\alpha) \ln x_2$ where $x_1 > 0$, $x_2 > 0$, and $0 < \alpha < 1$. His indirect utility function is $v(p, m) = \ln m - \alpha \ln p_1 - (1-\alpha) \ln p_2 + \ln K$ and his expenditure function is $e(p, u) = e^u p_1^{\alpha} p_2^{1-\alpha}/K$ where $K > 0$ is a constant. Note that in the expression e^u, the number e is the constant 2.718 . . .

(a) Carrying out appropriate calculations, show that x(p, m) = h[p, v(p, m)]. Interpret your results using a graph.

(b) Initially prices and income are $(p, m) > 0$. Due to a change in government policy, the prices change to $q \neq p$. What income m' does Jim need at the prices q in order to be exactly as well off as he was at (p, m)? After you solve this problem, briefly discuss what happens in the special case where $t > 0$ is a scalar and $q = tp$.

(c) The indirect utility function $z(p, m) = mK/p_1^{\alpha}p_2^{1-\alpha}$ is an increasing transformation of v(p, m). Assume all consumers i = 1 . . n have indirect utility functions of this form and let $M = \sum m_i$ be total income. Will it be possible to write the aggregate (market) demands in the form X(p, M)? Does it matter whether α_i differs across consumers? You don't need to calculate X(p, M) but give a careful explanation.

4. Joan has the utility function u(x, L) where $x \geq 0$ is food and $L \geq 0$ is leisure. Her only income is wH where $w > 0$ is the wage and $H \geq 0$ is her hours of work. The price of food is $p > 0$. Her time constraint is $H + L = T$ where $T > 0$ is total time.

(a) Suppose x and p are scalars. Write Joan's budget constraint in terms of the bundle (x, L). Then explain how you would solve for her demand for leisure. Finally use calculus and the Slutsky equation to show that her leisure L can increase when the price of leisure increases, even if leisure is a normal good. Explain carefully.

(b) Continue to assume x and p are scalars. Suppose $(x^*, L^*) > 0$ are given, and you want to find prices $(p, w) > 0$ such that Joan would choose (x^*, L^*). Describe a method you might use, and state any assumptions you need. Does your method work for all possible $(x^*, L^*) > 0$, or only for some? Discuss.

(c) Now let $x = (x_1 \, . . \, x_n) \geq 0$ be a vector of specific food items (rice, fish, etc.), and let $p = (p_1 \, . . \, p_n) > 0$ be the associated prices. Suppose you want to aggregate all of the food items into a single composite good called 'food' (big X). Describe a set of assumptions that permit you to do this (choose <u>one</u> of the two methods we covered in class; don't use both). Show how to construct a price for big X and a utility function U(X, L) involving big X. Carefully explain your reasoning.

5. There are n identical firms of type A with short run cost functions $c_A(y) = ay^2 + F$ and m identical firms of type B with short cost functions $c_B(y) = by + G$, where a, b, F, and G are all positive constants. Firms of type A can produce any output $y \geq 0$. Firms of type B are restricted to an interval $0 \leq y \leq y_{max}$ where $y_{max} > 0$.

(a) Describe the supply functions $y_A(p)$ and $y_B(p)$ for individual firms of each type mathematically and show these supply functions on separate graphs.

(b) Describe the market supply curve S(p) mathematically and show it on a graph.

(c) Using your graph from part (b), draw a market demand curve D(p) for each of the following cases: (i) only A firms have positive output; (ii) all firms have positive output but B firms do not produce y_{max}; (iii) all firms have positive output and the B firms would produce more output if they could. Explain your reasoning.